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THE LONGITUDINAL SHEAR STRENGTH REQUIRED IN DOUBLE-ANGLE COLUMNS OF 24S-T ALUMINUM ALLOY

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THE LONGITUDINAL SHEAR STRENGTH REQUIRED IN DOUBLE-ANGLE COLUMNS OF 24S-T ALUMINUM ALLOY

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SUMMARY

Tests were made of riveted double-angle columns to determine the total rivet strength that is required to make these built-up columns develop the strength predicted by the standard column formulas. Results of the tests led to the conclusion that the required rivet strength may be calculated by the beam method of design.

INTRODUCTION

It is well known that the load which a built-up column can carry is influenced by the shear stiffness of the column; the subject has attracted much attention since the failure of the Quebec bridge, and a considerable amount of theoretical work on the subject is recorded in engineering literature. Very little information is available, however, on the related problem of shear strength. A discussion of work published prior to 1920, both theoretical and experimental, is given by Salmon in his comprehensive treatise on columns (reference 1). The theoretical work is scanty and of necessity contains empirical coefficients. The experimental evidence is even scantier than the theoretical work and is confined to some strain measurements on the lattice bars of built-up columns. Recent tests of structurally similar columns (reference 2) appear to confirm reasonably well these earlier tests. Unfortunately, all the tests cover only a narrow range of slenderness ratios; furthermore, they were not carried beyond the range of working stresses used in civil-engineering practice; whereas aeronautical engineers are vitally concerned with ultimate stresses. The purpose of this paper is to present the results of an investigation of the shear strength required in a simple type of built-up column, namely, columns consisting of two angles riveted together.

TEST OBJECTS AND TEST PROCEDURE

The cross sections of the columns tested are shown in figure 1. Angles with unequal legs were chosen to insure failure in the desired direction. The individual angles were cut from 20-foot lengths of 24S-T aluminum-alloy angles and were riveted together with A17S-T rivets. Test coupons were cut from both ends of four of the 20-foot angles, and the stress-strain curves were determined. The grand-total average value of Young's modulus was 10.6×10^3 kips per square inch. The variation of individual moduli from the grand-total average was nearly $\pm 0.4 \times 10^3$ kips per square inch. Average moduli for the 20-foot lengths obtained by averaging the individual moduli for the two ends of each length, varied from the grand-total average by only $\pm 0.1 \times 10^3$ kips per square inch. The yield stress (0.2-percent offset) was 39.3 ± 1.6 kips per square inch. For the columns consisting of the smaller angles ($1 \times 5/8 \times 1/8$), only short lengths of angle were available; some of these angles were quite wavy, and no stress-strain curves were obtained for these small angles.

The ends of all column specimens were carefully milled flat and square. One series of columns was tested on knife-edge bearings; the majority of the columns, however, were tested flat-ended. The cross-sectional areas were determined by weighing the specimens.

METHOD OF ANALYSIS

Two different methods are commonly used to compute the necessary shear strength of built-up columns. One method consists in assuming that the transverse shear is 2 percent of the column load. (See, for instance, reference 3, p. 352.) The other method consists in assuming that the column is used as a beam, subjected to transverse loads with arbitrarily chosen distribution, and then using standard methods of beam design. Variations of this method take into account the column stress. (See, for instance, reference 4, p. 303.)

The origin of the 2-percent value is not entirely clear; it may be the tests of Talbot and Moore mentioned in reference 1. The longitudinal shear strength required by this method is

$$R_R = \frac{VQL}{I} = \frac{0.02\ PQL}{I} \qquad (1)$$

where

V transverse shear force, kips

R_R required total rivet strength (in single shear), kips

P column strength, kips

Q static moment of cross section of one angle about neutral axis of column, inches^3

I moment of inertia of cross section, inches^4

L actual length of column, inches

For the second method of computing the necessary shear strength of built-up columns, it is assumed that the two-angle structure is employed as a beam instead of a column; that is, it is subjected to transverse loads. If the column has pin ends, the beam is assumed to have simple supports. The transverse load is assumed to be symmetrical about the center line but may be distributed in any arbitrary manner. This load produces a maximum bending moment

$$M = V_{av} \frac{L}{2} \qquad (2)$$

where V_{av} is the average transverse shear force. The structure will fail when the maximum fiber stress produced by this bending moment reaches some limiting value F. The maximum fiber stress is

$$F = \frac{Mb}{I} = \frac{V_{av}Lb}{2I} \qquad (3)$$

where b is the width of the outstanding leg of one angle. The rivets must have sufficient strength to let the structure develop this fiber stress. The longitudinal shear strength required for this purpose is

$$R_R = \frac{V_{av}QL}{I} = \frac{2FQ}{b} \qquad (4)$$

If the column has fixed ends and the beam has correspondingly built-in ends, it is no longer possible to write a single formula analogous to formula (2) that is valid for any distribution of the transverse load. In order to avoid this difficulty, it will be assumed in this case that the load is concentrated at the middle. The transverse shear force is then constant, and a constant distribution is the most natural one to choose for developing a formula that gives the total longitudinal shear strength. Formula (2) becomes

$$M = \frac{VL}{4} \tag{2a}$$

and formula (4) becomes

$$R_R = \frac{VQL}{I} = \frac{4FQ}{b} \tag{4a}$$

There is no direct physical relation between the failure of a column and the failure of a beam that has the same cross section; the failing stress F to be used in formula (4) therefore is not necessarily the modulus of rupture of the material. It is a stress established, preferably, by working backward from tests such as those decribed in this paper. A large difference between the stress established in this manner and the modulus of rupture would indicate, however, that the method is questionable either as a whole or in part.

Formulas (1) and (4) were evaluated for the larger of the two columns tested. The column strengths needed for formula (1) were based on the standard column formulas given in reference 5 for 24S-T aluminum alloy

$$\sigma = \frac{103.8 \times 10^3}{(L'/\rho)^2} \text{ kips per square inch } \left(\frac{L'}{\rho} > 79.2\right) \tag{5}$$

$$\sigma = 50 - 0.431 \frac{L'}{\rho} \text{ kips per square inch } \left(\frac{L'}{\rho} < 79.2\right) \tag{6}$$

where L'/ρ is the effective slenderness ratio.

After a preliminary study of the test data, the value of F in formula (4) was taken as 50 kips per square inch. This stress is equal to the column yield stress for the material and may be considered a reasonable value for a failing stress in bending. The computed values of required rivet strength R_R for the larger columns are shown in figure 2. The required rivet strengths computed by the 2-percent method (formula (1)) vary considerably with slenderness ratio; whereas the strength computed by the beam method (formula (4)) is independent of the slenderness ratio.

TEST RESULTS

A preliminary inspection of the test data showed that the required rivet strength was practically independent of the slenderness ratio; formula (4) was therefore chosen as the basis for preparing figure 3, which shows the experimental relations between column strength and rivet strength for the slenderness ratios investigated. The value of F in formula (4) was taken as 50 kips per square inch. The effective slenderness ratio for the flat-end tests was taken as one-half the actual slenderness ratio.

Inspection of figure 3 shows that the faired curves for developed column strength lie, at most, 2 percent below the calculated column strength when the actual rivet strength equals the required rivet strength determined by formula (4). The curve for the flat-end test at an effective slenderness ratio of 17 crosses the 100-percent line when the actual rivet strength is only 0.24 of the required strength. At such small slenderness ratios, experimental column curves usually exhibit a "pick-up" that should be disregarded in design work (reference 5).

No flat-end columns developed 100 percent of the calculated strength except the shortest ones and one freak point at $L'/\rho = 61.5$. The difference, however, was only 1 percent in the columns that had adequate or very nearly adequate rivet strength and may have been caused by experimental inaccuracy or by failure to achieve full fixity. The columns tested with knife-edge bearings, on the other hand, developed slightly more than the calculated strength in every case in which adquate rivet strength was provided. The excess over the calculated strength may have been caused partly by the standard column formulas being somewhat conservative and partly by slight frictional moments in the bearings. In the main series of the tests (large angles with 3/16-in. rivets), the individual points in the flat-end tests show very little scatter. The larger scatter in the knife-edge tests may perhaps be attributed to variable friction in the knife-edge bearings.

The faired curves for the individual test series were averaged; the average curve, the highest curve, and the lowest curve are shown in figure 4. These curves may be used to estimate the loss of column strength caused by insufficient rivet strength. The curve pertaining to the test at $L'/\rho = 17$ was disregarded in the preparation of figure 4.

DESIGN FORMULAS

Formula (4) can be applied without difficulty when the ends of the column have either no fixity or full fixity. The application to other cases would be somewhat complicated. In view of the empirical nature of the formula, such complications are hardly warranted. It is therefore suggested that the formula be used in the form

$$R_R = 100 \frac{Q}{b} \sqrt{c} \tag{7}$$

where R_R is in kips, Q and b are in inch units, and c is the fixity coefficient, which lies between 1 and 4. The magnitude of F in formula (4) has here been taken as 50 kips per square inch.

It is probable that the formula can be used for aluminum alloys similar to 24S-T, except that it may become unconservative at low slenderness ratios for materials which have a column yield stress appreciably higher than 24S-T alloy (say, greater than 60 kips/sq in.).

In double-angle columns used as uprights on shear webs, the folds of the web will tend to split the two angles. Some allowance should be made for the resulting extra load on the rivets. A simple method of providing such an allowance would be to compute the fixity coefficient needed for formula (7) by the expression

$$c = 4 - \frac{2d}{h_e} \qquad \left(\frac{d}{h_e} < 1.5\right) \qquad (8)$$

where d is the spacing of the uprights and h_e the effective depth of the shear web. Formula (8) is the empirical relation for the fixity coefficient in a web under pure diagonal tension (reference 6); the fixity coefficient for pure diagonal tension is larger than that for incomplete diagonal tension, and the use of this higher coefficient in formula (7) gives a margin in the desired direction.

CONCLUSION

It is concluded that a double-angle column of average 24S-T aluminum alloy may be expected to develop 98 percent of the strength computed from the standard column curve. The total rivet strength required to develop this column strength can be calculated by the expression

$$R_R = 100 \frac{Q}{b} \sqrt{c}$$

where

R_R required rivet strength, kips

Q static moment of cross section of one angle about neutral axis of column, inches^3

b width of outstanding leg, inches

c fixity coefficient

The minimum pitch, of course, must be chosen to prevent buckling of the individual angles between rivets.

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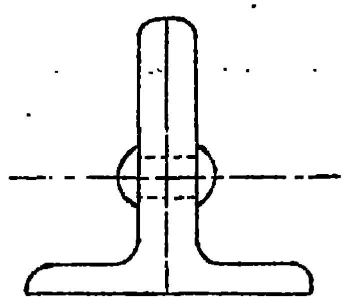

Figure 1.- Cross sections of test columns.

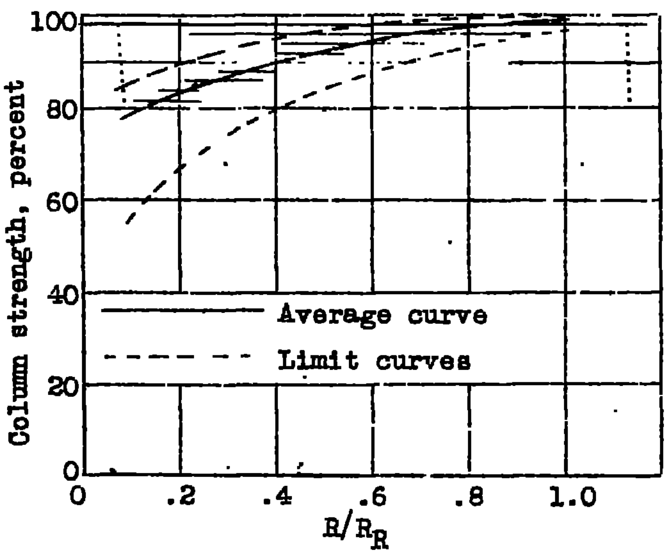

Figure 4.- Variation of column strength with rivet strength.

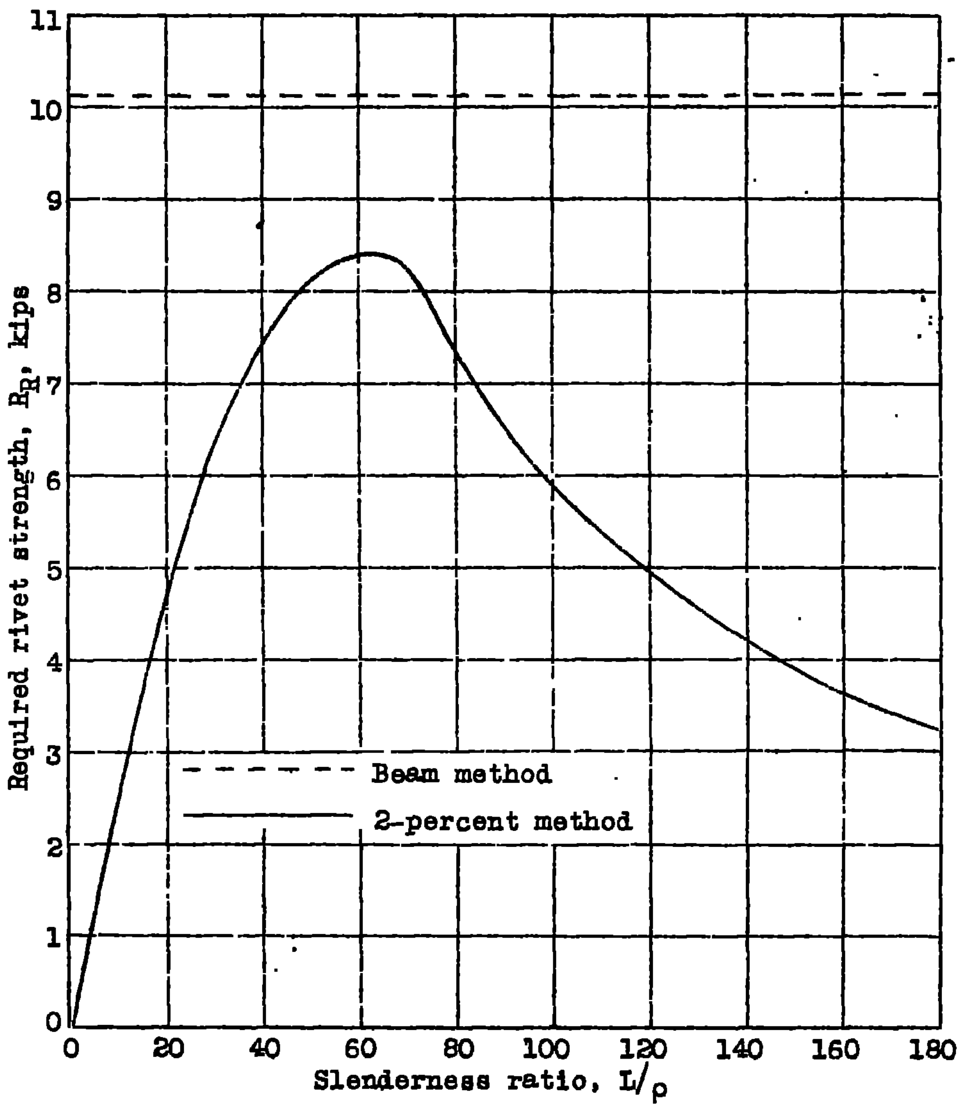

Figure 2.- Required rivet strength computed by beam method and by 2-percent method.

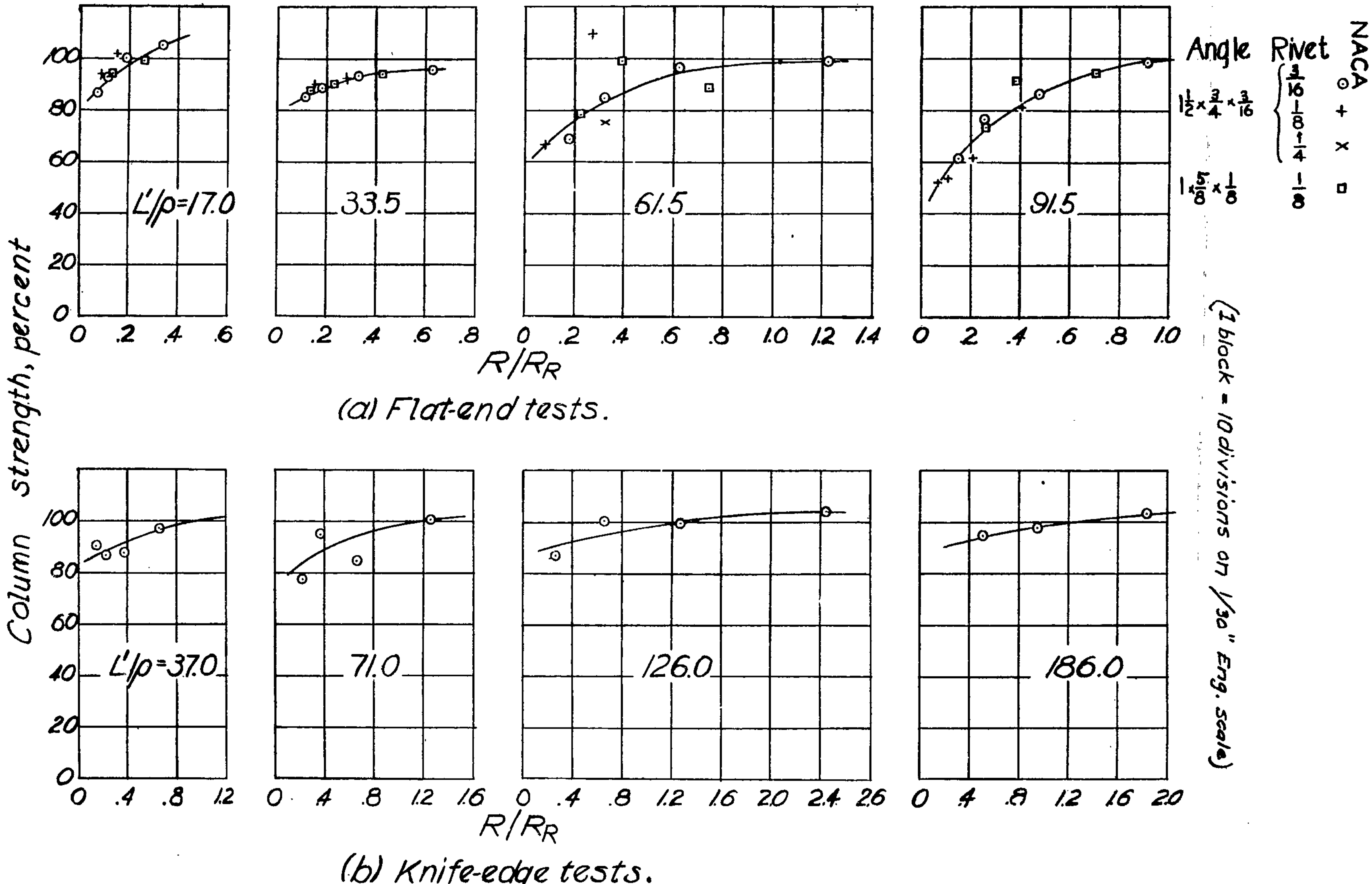

Figure 3.- Experimental variation of column strength with ratio of actual rivet strength R to required rivet strength R_R.

Fig. 3